

Shrinkage Theory for Portfolio Optimization with Correlated Geometric Brownian Motion

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Introduction

Since the beginning of the first stock exchange hundreds of years ago, analysts have been trying to create stock portfolios that provide them with the highest and most consistent returns. With improvements in quantitative and behavioral finance, analysts are now able to trade more frequently and remove many sources of human error. Portfolios have been optimized using equities, commodities, long and short positions, equally weighted portfolios, and many other strategies and financial instruments. During the 1950's, Harry Markowitz created a strategy for optimizing a stock portfolio focused on the simple goal of maximizing returns while minimizing variance. The only requirements for using Markowitz's technique are the returns of the stocks and a covariance matrix of these returns. Using computer technologies, Markowitz's mean-variance method is well understood throughout the financial industry and provides a quick strategy for determining stock positions. Despite the convenience of this technique, there are some very strong criticisms. One of the most important problems occurs when creating the set of means and covariance matrix, no one in the world knows the true mean return on any given stock nor can they be certain of the volatility for the returns. Because of this drawback, the covariance matrix can only be created using a sample of the stock returns available which results in significant estimation error. This paper will focus on the errors created by the sample covariance matrix, and how "shrinkage" can be used to reduce the error within this matrix. Including this question, there will be a secondary focus on discovering how much data is necessary in order to make accurate decisions about the future.

Instead of conducting the analysis on real data, which can involve many instances of market shocks or behavioral bias, a synthetic set of data will be created using Correlated Geometric Brownian Motion. The

simulation will create 35 years of daily returns (252 trading days per year), resulting in a robust data set for testing. A portfolio, known as the “Ground Truth” (GT), will be created as the benchmark for comparison with the of the Markowitz and Shrinkage portfolios. The GT portfolio will be created based upon all available “historical” data provided from the simulation. The analysis will be conducted based upon five key metrics: number of data points available (T), return, volatility, Sharpe ratio, and Active Share.

Background on Portfolio Optimization

Markowitz’s Mean-Variance Portfolio

Harry Markowitz’s concept of mean-variance analysis, in the general sense, is quite simple. The objective is to provide a portfolio with the minimum variance, for a desired level of return. This optimization problem has an infinite number of solutions and is illustrated symbolically below:

$$\begin{cases} \min: -t\mu'x + \frac{1}{2}x'\Sigma x \\ \text{s.t. } l'x = 1 \end{cases} \quad (1)$$

$$\mu = [\mu_1, \mu_2, \mu_3, \dots, \mu_n]'$$

$$\Sigma = [\sigma_{ij}] \quad i, j = 1, 2, 3, \dots, n$$

$$t \geq 0$$

$$x = [x_1, x_2, x_3, \dots, x_n]'$$

The parameters involved in (1) are the expected return μ of the n number of risky assets, the covariance matrix Σ (positive definite), the weight of the assets x , and the weight placed on the maximization of returns t . The larger the value of t , the larger the level of importance placed on of maximization of the returns, versus the minimization of variance. The single constraint on (1) is a weight constraint $l'x = 1$. The parameter x is the set of weights for each asset, and l is the vector of ones. For our purposes there is no constraint on short selling ($x_i < 0$). The concept of short selling involves borrowing an asset from Person A, and then selling that asset to Person B and an amount X_1 . At a date in the future, you pay Person A for the borrowed asset at the asset’s current value of X_2 . In the case of an asset that declines in value, there is a profit of $X_1 - X_2$. An investor will use short selling to raise capital, and invest that capital in other assets.

Efficient Frontier and Capital Market Line

In order to create a visual representation of the substitution between return and variance, the optimization problem must be computed for many values of t . Michael J. Best’s work in **PORTFOLIO OPTIMIZATION** (2010) provides a step-by-step process for solving the optimization problem. These many optimized portfolios

combine to create a parabolic function (Figure 1). As the figure shows, the variance of the portfolio is plotted on the horizontal axis and the portfolio's return is plotted on the vertical axis. The vertex of the parabola creates a portfolio of the given risky assets with the minimum variance. All points above the minimum variance portfolio are on the efficient frontier. While all points below the minimum variance portfolio are in the inefficient frontier, because there is a portfolio on the efficient frontier with the same variance but a higher return. Therefore, a portfolio manager will always choose a point on the efficient frontier, because all other options are either inefficient or impossible (those in the space above the efficient frontier) for the risky assets available.

In portfolio management, managers prefer having the option to diversify their portfolio among more than just a variety of risky assets (e.g. equities and commodities). A second asset class includes a risk-free asset x_{n+1} with a return of r (e.g. treasury bills – assumed to be completely risk-free). The optimization problem can now be re-written to include the risk-free asset:

$$\left\{ \begin{array}{l} \min: -t(\mu', r) \begin{bmatrix} x \\ x_{n+1} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x \\ x_{n+1} \end{bmatrix}' \begin{bmatrix} \Sigma & 0 \\ 0' & 0 \end{bmatrix} \begin{bmatrix} x \\ x_{n+1} \end{bmatrix} \\ \text{s. t. } l'x + x_{n+1} \end{array} \right. \quad (2)$$

All the same parameters from (1) are still in (2), but with the addition of the risk-free asset (covariance matrix is now positive semidefinite). Again, t is a key determinant of the portfolio. The only difference from equation (1) is when $t = 0$, the portfolio allocates all resources to the risk-free asset and none to the risky assets. The allocation of the risk-free asset decreases as the value of t increases. A more risk-averse portfolio manager will have a lower value of t . Eventually the value of t will increase to a point at which the allocation of resources in the risk-free asset reaches zero. The point at which this occurs, and all resources are allocated to the risky assets, will be the market portfolio. The values of t between zero and the market portfolio form a linear equation define by the difference between the expected return of the portfolio and the risk-free asset (3). This line is a function of the portfolio's standard deviation, the square-root of the variance σ_p^2 (4), which comes from the optimization problem (2).

$$\mu - r = \sigma_p [(\mu - rl)' \Sigma^{-1} (\mu - rl)]^{\frac{1}{2}} \quad (3)$$

$$\sigma_p^2 = x' \Sigma x \quad (4)$$

The line that combines all the portfolios, for different levels of variance, is known as the Capital Market Line (CML). The CML can be plotted in the same space as the efficient frontier, with the horizontal axis as the portfolio's standard deviation instead of variance (Figure 2). As seen in the plot of the efficient frontier and the CML, there is one point at which the CML is tangent to the efficient frontier. From our understanding of

the efficient frontier we know it is made up of only risky assets. Therefore the point of tangency must occur when t is large enough for the CML to no longer hold positions in the risk-free asset, the market portfolio.

With the addition of the CML to the efficient frontier, there is a larger range of potential portfolios to meet the needs of a more risk-adverse portfolio manager. A manager looking for a portfolio with less risk than the market portfolio, they will choose a point on the CML (i.e. solid line to the left of the market portfolio). Likewise a manager looking for a larger return than the market portfolio will choose a point along the efficient frontier (i.e. solid line to the right of the market portfolio). The dotted portion of the CML represents portfolios that are not possible with the weight constraint and assets available.

Markowitz Shortcomings

Harry Markowitz can be credited for creating an excellent platform for the study of portfolio optimization. **But the simplicity of the framework, can lead to serious issues when it is applied to real assets.** One of the key concerns of Markowitz's method, and a focus of this paper, is the **estimation error created with the sample covariance matrix.** **When computing the returns and covariance matrix, an analyst only has a finite number of observations to analyze.** Therefore, it is impossible for one to know the true mean return for any given asset. This means the covariance matrix being applied is only an estimation of what will happen in the markets. The estimation of the covariance matrix will then include a set of extreme values, and these extreme values will lead to extreme asset weights within the optimization of the portfolio. The significance of these values is multiplied when there are fewer constraints placed on the portfolio (e.g. no short sales, maximum of 10% allocated to one asset, etc.). As most portfolio managers will tell you, the best portfolio is a well-diversified portfolio. Therefore, placing a significant amount of your holding in a few substantial long and short positions is not a wise decision. The risk associated with the portfolio is much greater, due the chance of these extreme holdings not performing as expected (e.g. long positions resulting in a negative return or short positions providing a positive return).

Shrinkage Theory Applied to Markowitz Portfolio

The extreme bets that arise from Markowitz's studies of portfolio optimization can be a concern for any portfolio manager, and make them question their methods of investing. Since we know these extreme bets are a result of the large values in the covariance matrix, it would be best to reduce the magnitude of these values. High levels of positive error result from the largest values and high levels of negative error from the smallest values in the covariance matrix, resulting in large bets being placed on the assets associated with these values in the covariance matrix. **The objective of shrinkage is to pull these extreme values toward a central value.** The sophisticated part of shrinkage, studied by Ledoit and Wolf (2004), is to determine what this central

value should be (i.e. shrinkage target F), and how much pressure you put on the sample covariance (S) to reach this center (i.e. shrinkage intensity δ).

Sample Covariance vs Shrinkage Target

The tradeoff between the sample covariance and the shrinkage target is the amount of structure. The advantage of having very little structure, like the sample covariance matrix, is the ease of computing the matrix and the very minimal amount of bias. The disadvantages of this matrix, as discussed previously, are the high levels of estimation error. This disadvantage is most prevalent in situations where the number of assets under consideration is greater than the number of available observations for each asset. The opposite is true about the shrinkage target, which has a lot of structure. **The advantages are the minimal estimation error, and the disadvantages are the potential for biased estimations.** Therefore, the best solution is a covariance matrix ($\Sigma_{shrinkage}$), which **makes a compromise between the two levels of structure (5).** **The shrinkage intensity δ can take on values between zero and one.** Values closer to one imply high levels of structure, while values closer to zero imply low levels of structure.

$$\Sigma_{shrinkage} = \delta F + (1 - \delta)S \quad (5)$$

Computing the Shrinkage Target

In order to add structure to our sample covariance, we must create a matrix with that desired structure. Ledoit and Wolf (2003) suggest using a constant correlation matrix. Unlike the standard computation of covariance between two assets (6) which uses the two-dimensional correlation between asset 1 and 2, they suggest replacing that correlation with a universal correlation. The constant correlation is the average of all the correlations within a set of assets, and is used instead for the shrinkage target (7).

$$\sigma_{12} = \rho_{12}\sqrt{\sigma_{11}\sigma_{22}} \quad (6)$$

$$\bar{\rho} = \frac{2}{(N-1)N} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \rho_{ij}$$

$$f_{ii} = \sigma_{ii}$$

$$f_{ij} = \bar{\rho}\sqrt{\sigma_{ii}\sigma_{jj}}$$

$$F = \begin{bmatrix} f_{11} & \cdots & f_{1N} \\ \vdots & \ddots & \vdots \\ f_{N1} & \cdots & f_{NN} \end{bmatrix} \quad (7)$$

Computing the Shrinkage Intensity

More complicated than the shrinkage target, is the computation of the shrinkage intensity. The objective of the shrinkage intensity is to minimize the distance between the shrinkage target and the true

covariance matrix. In another work by Ledoit and Wolf (2003) they prove the shrinkage target has an asymptotic relationship similar to a constant κ over the number to observations T (8). Where the constant is computed from three other variables π , ϱ , and γ (9, 10, and 11, respectively).

$$\delta = \frac{\kappa}{T} \quad (8)$$

$$\kappa = \frac{\pi - \varrho}{\gamma}$$

$$\pi = \sum_{i=1}^N \sum_{j=1}^N \text{AsyVar}[\sqrt{T}\sigma_{ij}] \quad (9)$$

$$\varrho = \sum_{i=1}^N \sum_{j=1}^N \text{AsyCov}[\sqrt{T}f_{ij}, \sqrt{T}\sigma_{ij}] \quad (10)$$

$$\gamma = \sum_{i=1}^N \sum_{j=1}^N (f_{ij} - \sigma_{ij})^2 \quad (11)$$

Brownian Motion Simulation

Benchmark – Dow Jones Industrial Average

In order to create a market simulation with some level of real-world relevance, daily pricing data for the constituents of the Dow Jones Industrial Average (DJIA) was collected from 2000 to 2014 (3773 data points for each stock). This pricing data was used to create the starting prices, returns, volatility, and covariance between constituents of the DJIA. Using these statistics, the next step was to apply them to a Brownian Motion simulation to create the simulated data for analysis. **Brownian Motion is a stochastic process that can be used to simulate the ideas of the random walk**, hypothesized by Jules Regnault in the 19th century and further studied a few decades later by Louis Bachelier in the **THEORY OF SPECULATION** (2006). Bachelier determined that over small periods of time a stock's price would experience fluctuations, independent of previous fluctuations and the stock's value, around its intrinsic value. Most importantly, he noted these fluctuations were **normally distributed**.

Simple Brownian Motion

The most basic form of Brownian Motion is Simple Brownian Motion (SBM). To make SBM specific to stock returns, a component called drift is added to the process. The drift is the mean return value and a specified deviation. By looking at the formation of a SBM simulation (12 – 15), each value of X is based upon the preceding value as the time increases by increments of s . Each value of Z is a random variable, where $Z \sim N(0,1)$. Using SBM, each step will increase or decrease the previous amount by a specified value (e.g. $1+0.1+0.1-0.1=1.2$). This movement, does not increase or decrease like a compound return on a stock price (e.g. $(1.1)(1.1)(0.9)=1.089$). Also, each stock X is independent of every other.

$$s = \Delta t = t_2 - t_1 \quad (12)$$

$$X(t_1) = \sigma\sqrt{t_1}Z_1 + \mu t_1 \quad (13)$$

$$X(t_2) = X(t_1) + \sigma\sqrt{s}Z_2 + \mu s \quad (14)$$

$$X(t_k) = \sum_{i=1}^k (\sigma\sqrt{s}Z_i + \mu s) \quad (15)$$

Geometric Brownian Motion

In order to create a simulation that reacts to increases and decreases in price like a stock, the SBM needs to be converted to a Geometric Brownian Motion (GBM) process. The GBM process is the exponential version of SBM (16). Now, the simulation will produce a compound increase or decrease on the value of the previous time period. But still, each stock in the market will act independently of every other.

$$S(t) = S(t - 1)e^{X(t)} \quad (16)$$

Correlated Geometric Brownian Motion

Now that there is a simulation to compound stock prices, they need to react to one another. The simulation needs to represent a specified level of correlation between each stock. This correlated simulation is accomplished using Correlated Geometric Brownian Motion (CGBM). Simple stated, CGBM makes multiple random variables move together based upon a certain level of correlation. This simulation is built off of GBM in (16). The major difference from GBM, is to combine the random variable Z with a Cholesky decomposition C of the covariance matrix Σ for the group of stocks (18). This combination forms a new random variable V . Discovered by Andre-Louis Cholesky, the Cholesky decomposition is the upper triangular matrix of a symmetric positive-definite matrix which satisfies (17). Using the new random variable V , the CGBM for two different stocks becomes (19 & 20).

$$C^T C = \Sigma \quad (17)$$

$$V = C^T Z \quad (18)$$

$$S_{t+s}^A = S_t^A e^{((\mu_A - \sigma_A^2)s + \sqrt{s}V_A)} \quad (19)$$

$$S_{t+s}^B = S_t^B e^{((\mu_B - \sigma_B^2)s + \sqrt{s}V_B)} \quad (20)$$

Now the concept of CGBM is understood, it can be applied to the simulation of a financial market where there are multiple correlated stocks reacting to each other over a long time period. For this paper, there were 30 individual stocks that were simulated over a period of 35 years (8820 daily prices). The results to this simulation are found in Figure 3.

Metrics for Analysis

Data Points Available (T)

The way the results of a portfolio change as T increases is one of the major concerns of this paper. Portfolios created based on 3 months, 6 months, 1 year, 3 years, 5 years, and 10 years will be shown specifically throughout this paper (63, 126, 252, 576, 1260, and 2520 days, respectively). T will also be represented as a continuous change over the range of 3 months to 10 years.

Rate of Return

The rate of return r is one of the simplest measures of financial data. It depicts the way the price of a stock has changed over a period of time (21). Throughout this paper, the rate of return will be discussed in either daily or annual terms. In order to compute the return of a portfolio R , which includes separately weighted securities, one must sum the weighted-return of each security in the portfolio (22).

$$r = \frac{P_1 - P_0}{P_0} \quad (21)$$

$$R = \sum_{i=1}^n w_i r_i \quad (22)$$

Volatility

Now it's nice to know what return a portfolio of stocks can receive, but it's more important to know how probable that return really is. This probability is volatility σ , sometimes it's also referred to as the standard deviation. This measure is computed from the square root of the variance σ^2 , which is a measure of the deviation of returns from the average return (23). The volatility of a portfolio is the sum of the combination of products of two asset weights and the covariance between those two assets (24).

$$\sigma = \sqrt{E[(r - \mu)^2]} \quad (23)$$

$$\sigma_P = \sum_{i,j=1}^n w_i w_j \sigma_{ij} \quad (24)$$

Sharpe Ratio

Even better than having a rate of return and a volatility for any security or portfolio of securities, is a metric that accounts for both measures. This single metric is the Sharpe Ratio, which measures the risk-adjusted return against a risk free rate of return r_f (25). The greater the Sharpe Ratio, the better return one is receiving for the amount of risk they are competing against.

$$\text{Sharpe Ratio} = \frac{r - r_f}{\sigma} \quad (25)$$

Correlation Matrix Distance (CMD)

Based on the work of Herdin, Czink, Ozcelik, and Bonek (2005), there is a convenient way to measure the change that has occurred to the correlation matrix when the shrinkage methods have been applied. This is the CMD, which is calculated using (26). The numerator of the quotient is the trace of the product between two correlation matrices, and the denominator is the product of the norm of each matrix. The one assumption to calculating the CMD, is that the data is non-stationary. The correlation matrices, which are based off of the daily returns, are stationary data series. The data points for each security revolve around a mean of zero and fluctuate without any sign of a trend, making the data stationary. The conversion of the covariance matrix to its correlation matrix can be seen in (27).

$$CMD = 1 - \frac{tr\{Corr_1 Corr_2\}}{\|Corr_1\| \|Corr_2\|} \quad (26)$$

$$Corr = \sqrt{diag(\Sigma)} * \Sigma * \sqrt{diag(\Sigma)} \quad (27)$$

Active Share

Active Share is a second metric for distance, similar to CMD. Active Share was developed by Cremers and Petajiso as a way of quantifying active portfolio management (2006). Unlike CMD, it's a measurement more commonly used in the financial industry. In the professional world, an analyst would use Active Share to decide whether or not the fees a fund manager is requesting are worth investing in that fund versus an index. This metric describes how a weights of a portfolio differ from a selected benchmark index. Active Share is described simply by (28), where the sum is compute of the number of assets i to N .

$$Active\ Share = \frac{1}{2} \sum_{i=1}^N |weight_{fund,i} - weight_{index,i}| \quad (28)$$

Analysis of Results

*Throughout the remainder of the paper, all references to a Markowitz portfolio will be seen as blue in the plots, Shrinkage portfolios in red, and anything referring to the GT portfolio will be in green.

Determining Ground Truth

Before proceeding with the analysis, it's important to understand the GT portfolio for which all other will be compared to. The GT portfolio is a standard Markowitz portfolio where $T = 30$ years (7560 daily prices). The GT portfolio has all possible historical data from the CGBM simulation. The Markowitz portfolio was chosen because there is very little difference between it and the Shrinkage portfolio (Active Share = 0.0103).

Efficient Frontier vs. Size of Data Set

One of the main hypothesis behind any data analysis is that the larger the data set becomes, the more accurate and predictable future results will be. Therefore, if more historical data on a financial security is available, then one's understanding of its behavior should be greater and the predictions of future behavior should be more accurate. Looking at Figures 4 and 5, which show efficient frontiers at increasing levels of T, one can see that as T increases, the efficient frontier becomes smaller. One interesting finding from both graphs, is that the expected return and volatility of the market portfolio (where 100% is in risky assets and 0% in the risk free) decrease as T increases from 3 months to 10 years. Intuitively, one may hope to find a higher expected return with lower volatility, because T is higher and therefore should result in more intelligent results. This theory makes one remember the portfolios on the efficient frontier are generated only based on the inputs to a quantitative model. In this case, the short period of 3 months (63 days) can include sharper trends. Two examples would be a consistent negative trend which would result in a large short position, or a consistent positive trend which would result in a large long position from the algorithm. We recognize from Tables 1 and 2, the greatest long and short positions are seen in the first two levels of T (3 and 6 months) for either the Markowitz or Shrinkage portfolios. The figures also show a convergence of the Efficient Frontier in both cases. The portfolios for T at 3, 5, and 10 years appear to overlap, implying the weights within each portfolio are very similar.

Return, Volatility, and Sharpe Ration vs. Size of Data Set

By Looking as Figure 6, one can gather that there is a stabilization in the return, volatility, and Sharpe Ratio between the Markowitz and Shrinkage portfolios. These figures represent annual values of these three metrics for 1 year of future returns (252 days, year 30-31 of simulation). This aligns with the findings of the Efficient Frontier analysis in the previous section, in which there was very little change after the 3 year mark. The convergence of the Markowitz and Shrinkage portfolios can easily be explained by Figure 7. By looking at the trend of the Shrinkage Intensity as T increases, one can see that it approaches zero, meaning the majority of the weight from Shrinkage is placed on the original covariance matrix that was used in the Markowitz portfolio and not the Shrinkage Target.

There is one rather strange phenomenon occurring in the Figure 6. This phenomenon occurs while T is less than approximately 500 (close to 2 years). There is a very noticeable shock to the system in which the return and Sharpe Ratios plummet, and the volatility spikes (Tables 3 & 4). This may be due to a "stock market crash" early in the data set of future prices, or it may just be a result of there not being enough data (T too small) to conduct an accurate analysis of past prices to predict accurate future portfolios. This will be looked at very shortly.

Active Share – Benchmarked Against GT Portfolio

Looking at Figure 8, the Active Share values for the Markowitz and Shrinkage portfolios are benchmarked against the GT portfolio for levels of T from 3 months to 10 years. Again, the trend of convergence continues. Both the Markowitz and Shrinkage portfolios converge to an Active Share value close to one. This shows the weights within the portfolios are approaching those of the GT portfolio. Also, the Active Share for the Markowitz and Shrinkage portfolios converge toward one another as T increases. This follows along with the finding of the other metrics (return, volatility, and Sharpe Ratio). Looking at the difference between the Active Share values of the Markowitz and Shrinkage portfolios, especially at low levels of T, there is a lot of fluctuation there as well. These fluctuations show that the two portfolios are not weighted in a similar manner for the early levels of T, which again promotes the phenomenon of T under two years.

Unusual Circumstances (Early Shocks)

As it was mentioned earlier, there appears to be a market shock when T is less than 2 year (504 data points). The question remains though, whether this shock is due to the data for those years or an inefficiency in the portfolio optimization practices for the Markowitz and Shrinkage portfolios. One way of determining this is by adjusting the starting point for the “future” returns. Currently this point is after the first 30 years of simulated data (7560 data points). That will now be moved back by two years (504 data points), making the new starting point at 28 years (7056 data points). Prior to moving this starting point, Figure 6 shows that the statistics begin to flatten around T = 500. Therefore, if there remains a shock to the system, it will appear in the adjusted version of those figures and keep them from flattening out at that point. The analysis will still have T proceed from 3 months (63 data points) to 10 years (2520 data points).

After adjusting the starting point, Figure 9 and Tables 5 & 6 represent how the portfolios have reacted. There appears to still be a very dramatic shock to the system, although not entirely in the same manner. The Markowitz portfolio experiences a spike in returns, and even remains above the GT portfolio returns for the adjusted period of T. While the Shrinkage portfolio experiences a more severe drop in returns during the early level of T. Looking at the volatility, both portfolios experience significantly higher levels. Probably the most surprising result after looking at the return and volatility over the increasing levels of T, is that the Sharpe Ratio has a tremendous boost in performance. There is a continuous increases in the Sharpe Ratios, and the Markowitz portfolio maintains a superior Sharpe Ratio until the portfolios converge with the GT portfolio value. By comparing the standard and adjusted versions of analysis side by side, one can see that the shock to the adjusted system does not last as long, tapering out close to the 1 year mark (252 data points). Moving along the adjusted graph to the marking for T = 500 (close to the old starting point), there is no longer the same shock to the statistics. This implies the shock had less to do with the data acting unpredictably around this point in

time. What it does imply, is there is a large potential for error when one does not have a large amount of historical data when attempting to plan for future financial success.

Conclusion

The objective of this paper was to focus on the errors created by the sample covariance matrix within Markowitz portfolio optimization, and how “shrinkage” can be used to reduce the error within this matrix. Including this question, there was a secondary focus on finding out how much data is necessary in order to make accurate decisions about the future. This was accomplished by first creating a simulation of the financial markets, in particular the DJIA, by using CGBM. Using the returns from this simulation, the data was tested using the Markowitz and Shrinkage portfolios and compared using the rate of return, volatility, Sharpe Ratio, and Active Share. These metrics, and other visual interpretations, were compared against the GT portfolio. The GT portfolio was a Markowitz portfolio with the maximum amount of historical data available.

Markowitz vs. Shrinkage

After computing the two portfolios and comparing them across many quantitative and qualitative values, there is no sufficient evidence to conclude the Shrinkage portfolio produces better results. In particular, there is no evidence that the Shrinkage portfolio reduces the error when compared to the Markowitz portfolio. In fact, some early levels of T had larger long and short positions for the Shrinkage portfolio than the Markowitz portfolio.

Performance as T Increased

Following along with the second question of the report, each form of analysis was reviewed for trends that occurred as T increased. The analysis found that there was a significant impact as the value of T increased. In particular, the metrics began to stabilize and converge toward the values of the GT portfolio. This allowed us to conclude that as T increased, the predictability of the market improved. This makes intuitive sense, because when one has more data at their disposal, they should be able to make more accurate decisions.

Future Adjustments

Although there is no solid evidence in this paper to prove that the Shrinkage portfolio is better than the Markowitz portfolio, there is always room to extend the analysis even further. The goal of this paper was to analyze this question in a concise and effective manner, which was accomplished by limiting the analysis to a few very well-known metrics (T, return, volatility, Sharpe Ratio, and Active Share). But in the future more adjustments could be made, especially to the constraints on the portfolios. This paper essentially added zero

constraints to the weighting of the portfolio, in the hope of being able to pick out larger sources of error. But it may be more practical in the future to adjust the portfolio to add constraints such as no shorting or all weights must be less than 10 percent, and these constraints may adjust the findings. There is also the matter of whether or not these portfolios would work with real financial data. This paper took a more mathematical approach to the problem, by focusing on the covariance matrices. But in reality, the ultimate goal of the portfolio optimization problem is to compute efficient allocations of real securities. Also, a majority of successful portfolio manager do not implement a buy and hold strategy. They are always adjusting the weights of their portfolios to account for new trends in the markets. Future studies relating to this paper could benefit from applying supplemental trading algorithms such as moving average strategies, options trades, and many others. On the whole, this paper addressed the concepts of Markowitz portfolio optimization and the applications of shrinkage, and made accurate conclusions based on the desired complexity of the study, but there will always be room to produce a more complex study of the topic.

Lastly, all who strive to understand Harry Markowitz's portfolio optimization methods should think very hard about what they are trying to accomplish. Even the Nobel Prize winner himself was still not convinced of his own methods even decades later when he was discussing the 1987 stock market crash.

"I should have computed the historical covariances of the asset classes and drawn an efficient frontier... I split my contributions 50/50 between stocks and bonds."

– Harry Markowitz

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Appendix 1 – Figures

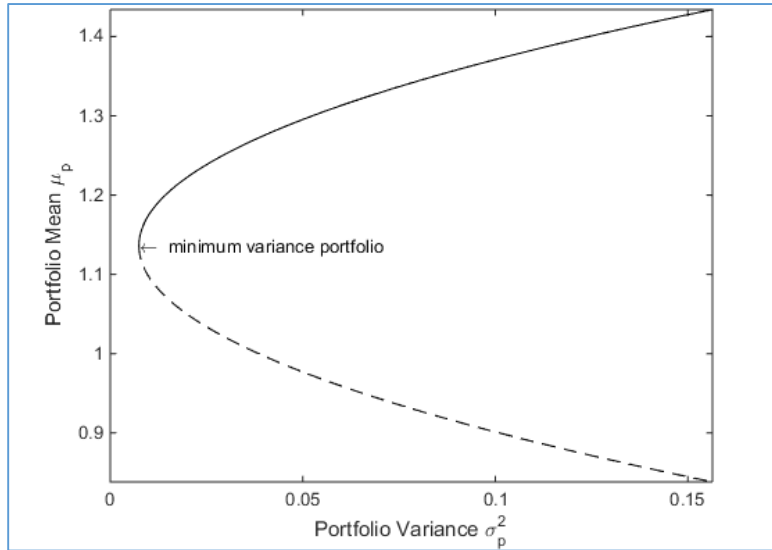


Figure 1: Efficient Frontier

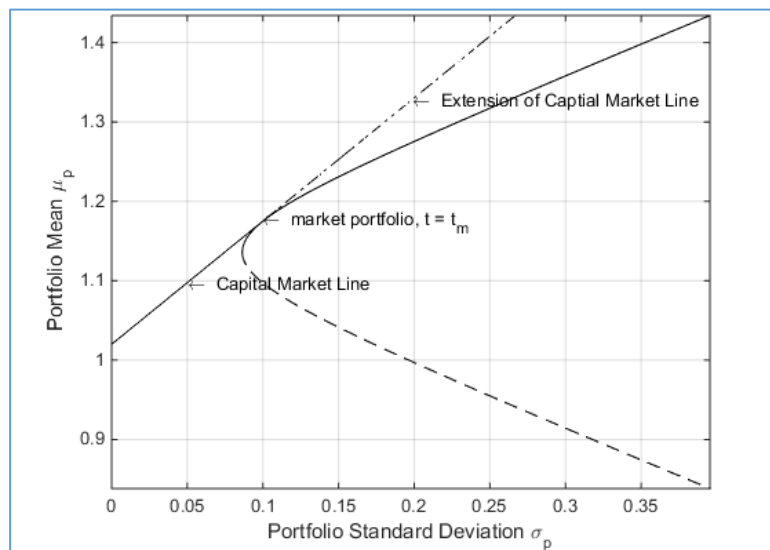


Figure 2: Efficient Frontier with Capital Market Line

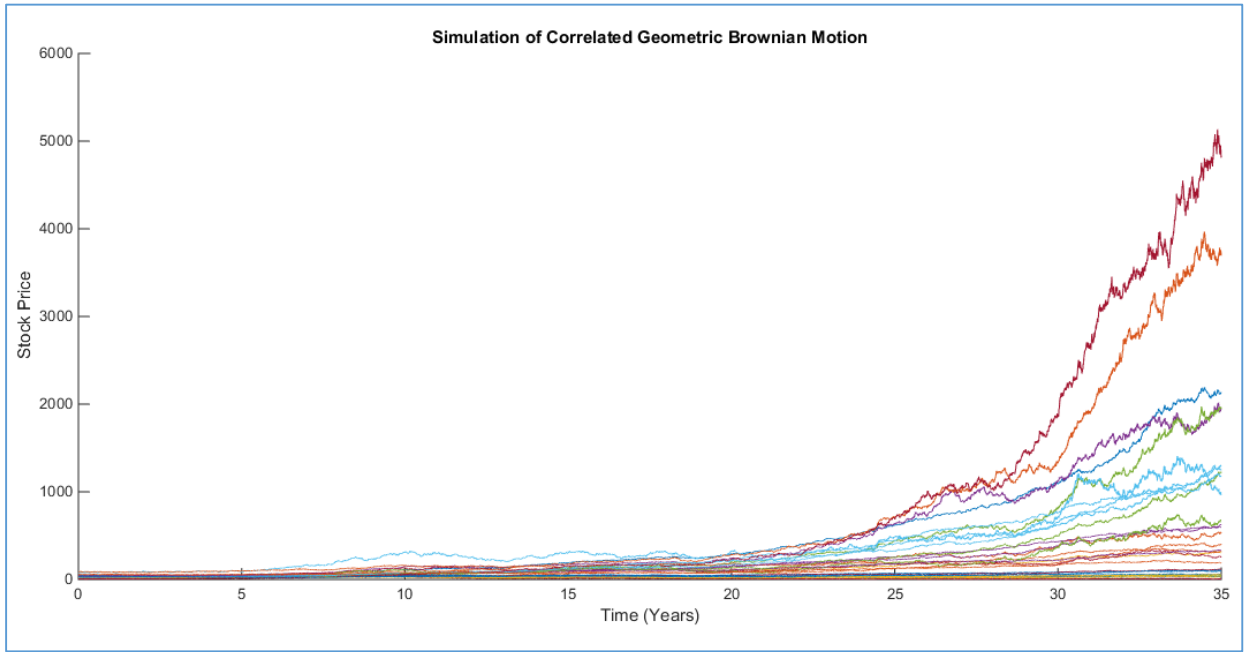


Figure 3: Simulation of Correlated Geometric Brownian Motion

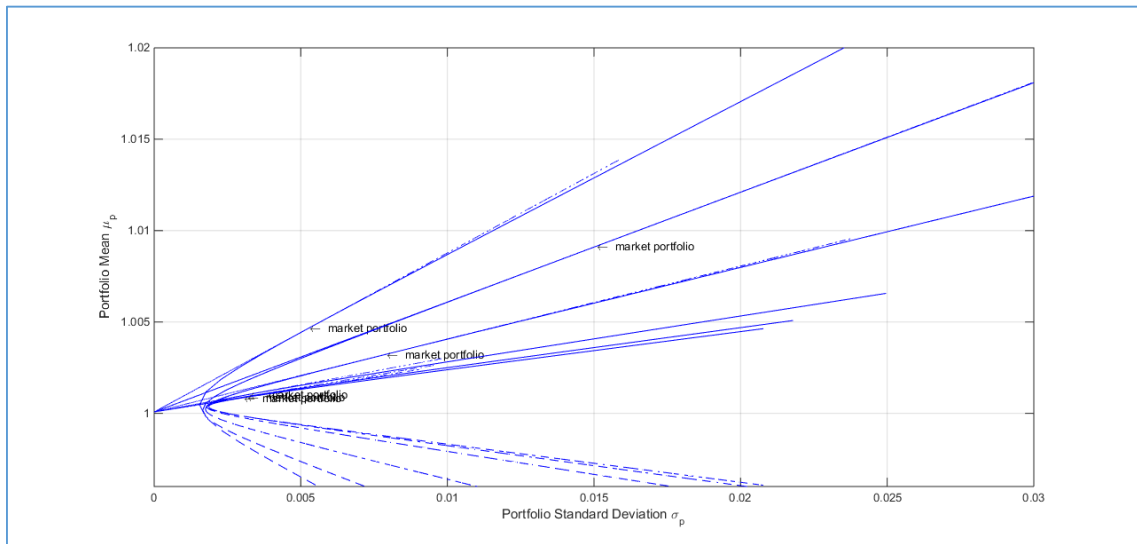


Figure 4: Markowitz Efficient Frontier & CML as T Increases (3 months to 10 years)

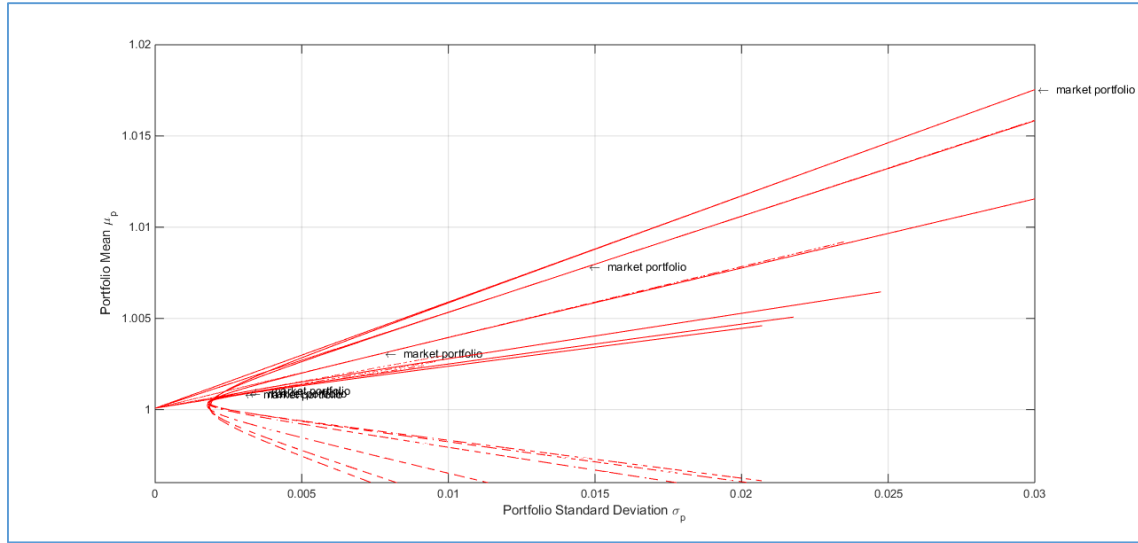


Figure 5: Shrinkage Efficient Frontier & CML as T Increases (3 months to 10 years)

Markowitz Portfolio Weights							
Stock	3 Months	6 Months	1 Year	3 Years	5 Years	10 Years	GT
1	-0.0678	0.8375	0.1142	0.3655	0.3643	0.4000	0.5122
2	-0.3358	-0.8202	0.1389	-0.0467	-0.0808	-0.0341	-0.0464
3	0.4818	1.8721	0.3410	0.1111	0.0903	-0.1178	-0.1711
4	-0.1464	-0.3811	0.0816	-0.0535	0.0341	0.0596	0.0670
5	0.0679	0.8541	0.3816	0.0212	0.0137	0.0363	0.0734
6	1.1636	2.0759	0.3486	0.2910	0.3586	0.1633	0.2293
7	-0.6207	-0.9094	-0.3368	-0.0625	-0.0673	-0.0816	-0.0846
8	0.2121	-1.6969	-0.7847	-0.0679	-0.1895	-0.1611	-0.1394
9	0.0245	-0.6724	-0.3227	-0.0687	0.0116	-0.0071	-0.0046
10	-0.0319	1.0232	0.0502	-0.3362	-0.0925	-0.1725	-0.2321
11	-1.0049	-1.1314	0.4243	0.0719	-0.1404	-0.0964	-0.0525
12	0.1129	0.4829	-0.5154	-0.1805	-0.1535	-0.1476	-0.2385
13	-0.1215	-0.3809	-0.0941	-0.0264	-0.0134	0.0048	0.0224
14	0.0289	-0.6395	-0.5954	-0.1623	-0.1347	-0.0755	-0.0870
15	0.7223	1.0720	0.2696	0.0266	0.0163	-0.0312	-0.0403
16	0.0333	0.0947	0.2709	0.2277	0.1231	0.0321	0.0640
17	-0.0926	0.1911	-0.1075	-0.0041	-0.0043	0.0124	0.0068
18	-0.7243	-0.4508	0.7640	0.8099	0.8946	0.8975	0.8009
19	0.0085	-0.1552	0.1539	0.1344	0.0535	0.0373	0.0327
20	0.3139	-0.8568	0.2077	0.1739	0.2635	0.2989	0.3368
21	0.2480	0.7831	0.2483	-0.0557	-0.1178	-0.1363	-0.1744
22	0.3125	0.9186	0.3895	-0.0308	-0.0183	-0.0585	-0.0706
23	0.3048	-0.1099	-0.0849	-0.0017	0.0800	0.1189	0.2127
24	-0.5943	-1.4179	-0.8572	-0.2170	-0.2146	-0.3058	-0.2606
25	-0.3624	-0.7294	-0.5984	-0.2341	-0.1404	-0.0021	-0.0062
26	-0.0109	-0.0860	0.1282	0.1149	0.0803	0.0401	0.0255
27	0.1743	0.4304	0.3711	0.0005	-0.0623	0.0996	0.1125
28	0.1996	0.5099	0.2224	0.0683	0.0894	0.1064	0.1386
29	-0.0186	-0.4098	-0.0175	-0.0865	-0.1445	0.0819	0.0342
30	0.7232	0.7021	0.4086	0.2176	0.1010	0.0384	-0.0606
Max Long	1.1636	2.0759	0.7640	0.8099	0.8946	0.8975	0.8009
Max Short	-1.0049	-1.6969	-0.8572	-0.3362	-0.2146	-0.3058	-0.2606

Table 1: Markowitz Portfolio Weights

Shrinkage Portfolio Weights							
Stock	3 Months	6 Months	1 Year	3 Years	5 Years	10 Years	GT
1	-0.3984	0.3076	0.0948	0.3562	0.3611	0.3988	0.5122
2	-0.3925	-0.4923	0.1159	-0.0395	-0.0758	-0.0322	-0.0464
3	1.0309	1.0278	0.2827	0.1039	0.0820	-0.1164	-0.1711
4	0.0736	-0.1553	0.0790	-0.0579	0.0330	0.0607	0.0670
5	0.0600	0.8081	0.3378	0.0144	0.0146	0.0361	0.0734
6	3.6368	1.3548	0.4171	0.2779	0.3325	0.1558	0.2293
7	-1.4496	-0.6682	-0.3096	-0.0671	-0.0688	-0.0818	-0.0846
8	0.8811	-1.1719	-0.7660	-0.0791	-0.1948	-0.1608	-0.1394
9	0.0022	-0.2835	-0.2521	-0.0734	0.0103	-0.0070	-0.0046
10	0.5554	0.6709	0.0982	-0.3178	-0.0872	-0.1683	-0.2321
11	-0.3853	0.3262	0.4849	0.1087	-0.1021	-0.0855	-0.0525
12	0.6947	0.1989	-0.4820	-0.1759	-0.1528	-0.1452	-0.2385
13	-0.3116	-0.3245	-0.0586	-0.0189	-0.0121	0.0040	0.0224
14	0.7029	-0.5699	-0.5189	-0.1620	-0.1361	-0.0754	-0.0870
15	1.7850	0.7617	0.2457	0.0271	0.0156	-0.0317	-0.0403
16	0.6794	0.3733	0.2848	0.2284	0.1211	0.0306	0.0640
17	-0.3454	0.1905	-0.0844	-0.0040	-0.0067	0.0099	0.0068
18	-4.1035	-0.5375	0.7395	0.8122	0.8883	0.8935	0.8009
19	-0.0607	-0.1208	0.1156	0.1217	0.0483	0.0351	0.0327
20	0.2325	-0.8895	0.2163	0.1726	0.2660	0.2989	0.3368
21	1.2429	0.6102	0.1979	-0.0580	-0.1173	-0.1369	-0.1744
22	1.0435	0.7140	0.3056	-0.0351	-0.0209	-0.0593	-0.0706
23	0.4054	-0.2492	-0.1290	-0.0025	0.0813	0.1193	0.2127
24	-2.6306	-1.1254	-0.7511	-0.2100	-0.2126	-0.3038	-0.2606
25	-4.3114	-0.8493	-0.6304	-0.2435	-0.1418	-0.0017	-0.0062
26	-0.3465	-0.2398	0.1121	0.1218	0.0812	0.0407	0.0255
27	-0.1848	0.1151	0.3229	0.0015	-0.0571	0.1001	0.1125
28	0.4353	0.4665	0.2248	0.0741	0.0929	0.1074	0.1386
29	0.3145	0.3825	0.0500	-0.0756	-0.1380	0.0776	0.0342
30	2.1442	0.3688	0.2566	0.1998	0.0956	0.0376	-0.0606
Max Long	3.6368	1.3548	0.7395	0.8122	0.8883	0.8935	0.8009
Max Short	-4.3114	-1.1719	-0.7660	-0.3178	-0.2126	-0.3038	-0.2606

Table 2: Shrinkage Portfolio Weights

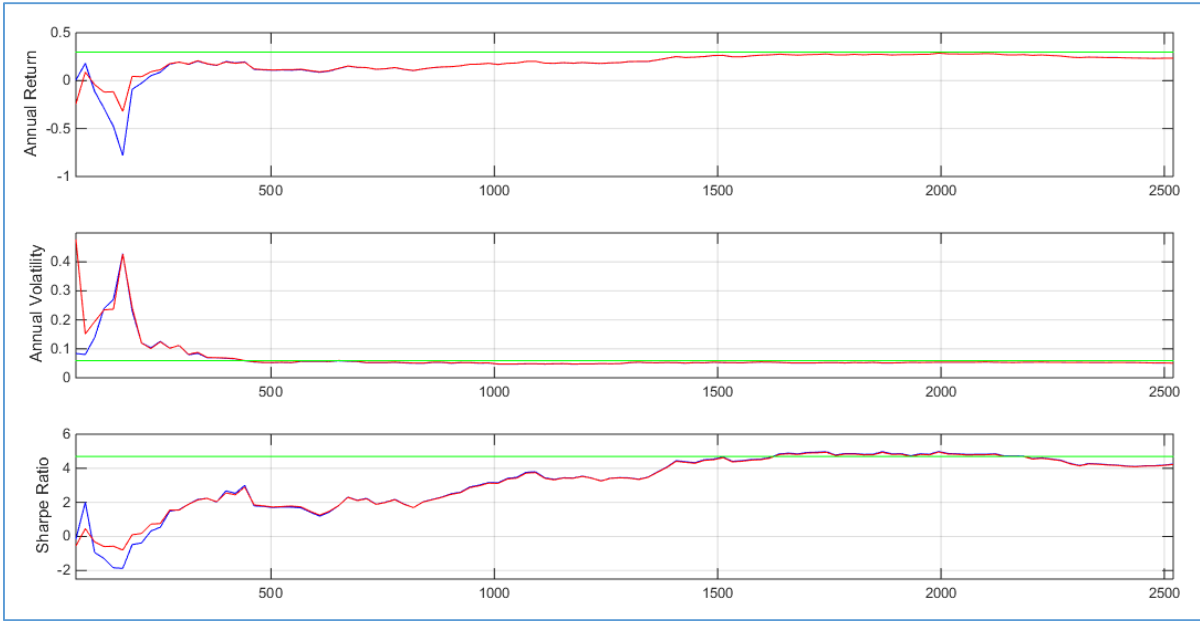


Figure 6: Return, Volatility, and Sharpe Ratio as T Increases (3 months to 10 years)

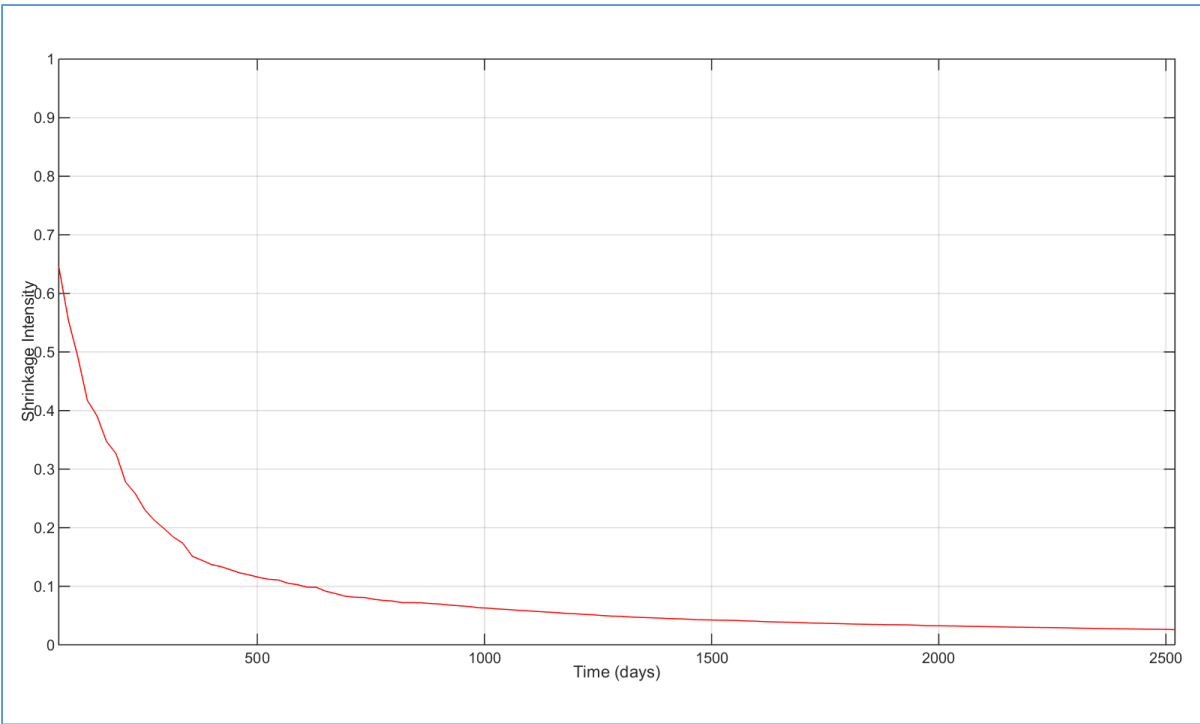


Figure 7: Shrinkage Intensity as T Increases

Markowitz Portfolio Statistics							
T	3 Months	6 Months	1 Year	3 Years	5 Years	10 Years	GT
Return	0.0065	-0.2887	0.0882	0.1242	0.1851	0.2348	0.2980
Volatility	0.0840	0.2393	0.1258	0.0519	0.0483	0.0506	0.0592
Sharpe	-0.1606	-1.2901	0.5421	2.0081	3.4204	4.2471	4.6953

Table 3: Markowitz Portfolio Statistics

Shrinkage Portfolio Statistics							
T	3 Months	6 Months	1 Year	3 Years	5 Years	10 Years	GT
Return	-0.2432	-0.1193	0.1143	0.1251	0.1860	0.2342	0.2980
Volatility	0.4776	0.2345	0.1243	0.0528	0.0487	0.0507	0.0592
Sharpe	-0.5512	-0.5938	0.7587	1.9912	3.4100	4.2263	4.6953

Table 4: Shrinkage Portfolio Statistics

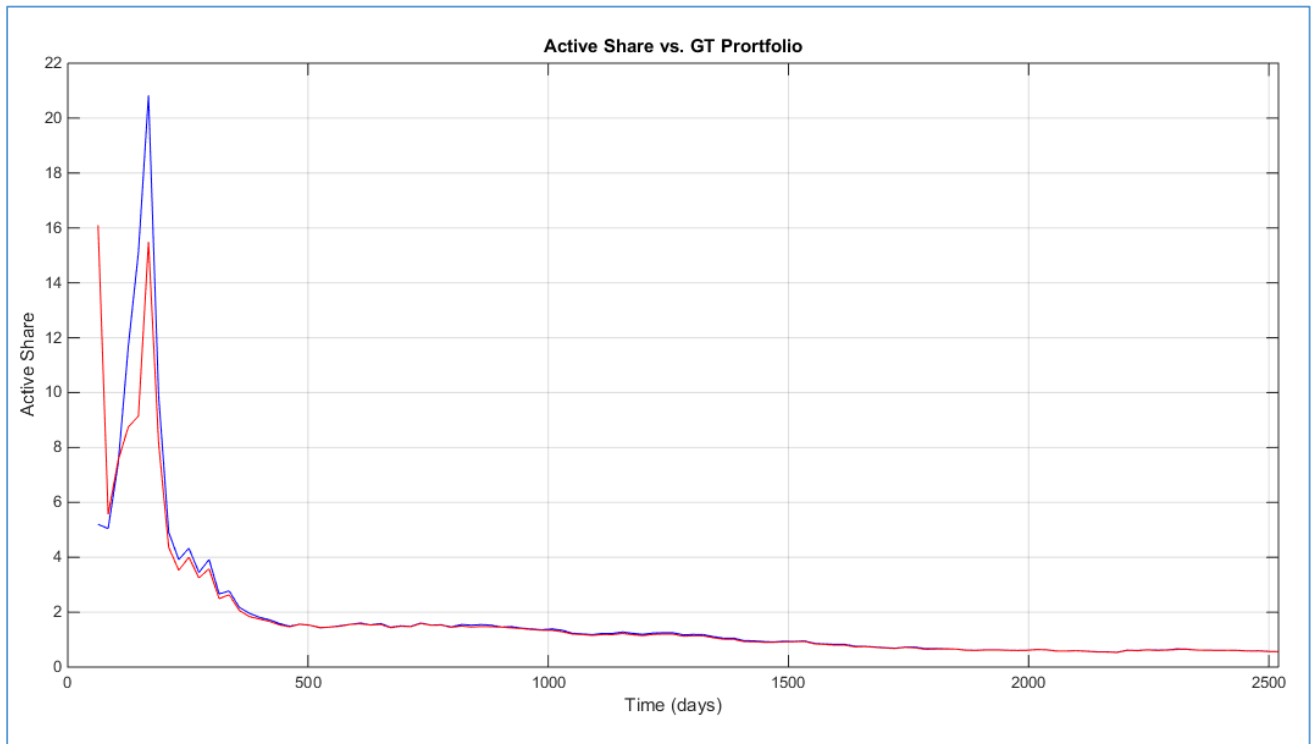


Figure 8: Active Share vs. GT Portfolio

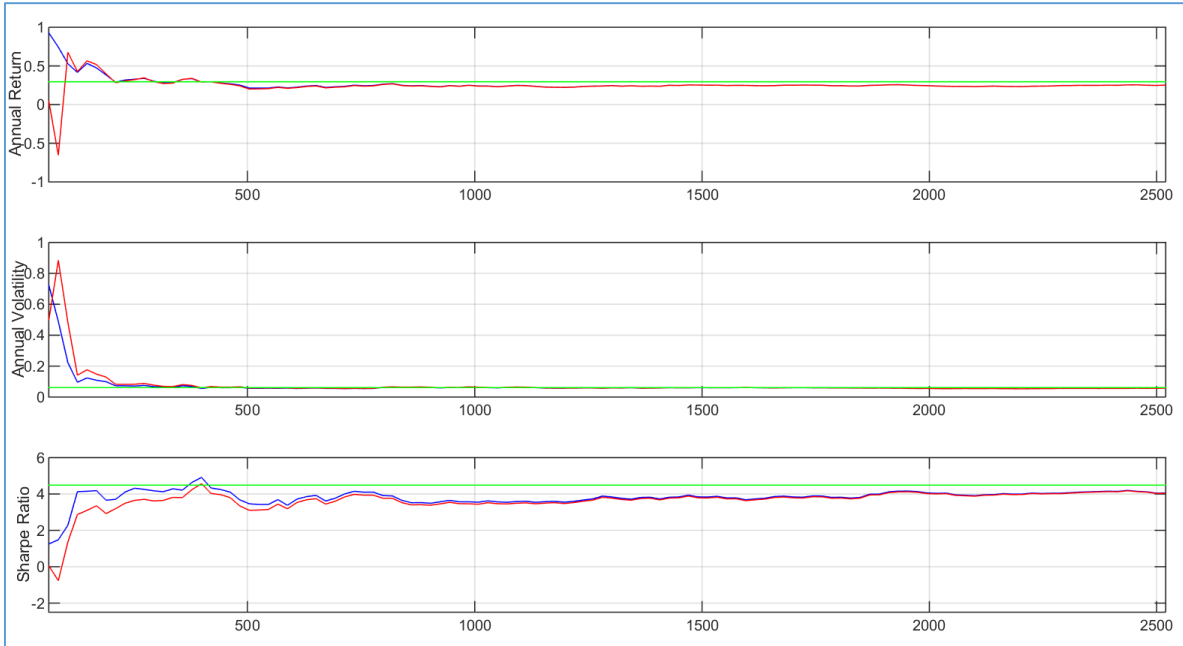


Figure 9: Adjusted Statistics for Return, Volatility, and Sharpe Ratio as T Increases (3 months to 10 years)

Adjusted Markowitz Portfolio Statistics							
T	63	126	252	756	1260	2520	7056
Return	0.9294	0.4157	0.3279	0.2438	0.2385	0.2523	0.2948
Volatility	0.7252	0.0959	0.0713	0.0546	0.0585	0.0573	0.0613
Sharpe	1.2539	4.1287	4.3167	4.0970	3.7332	4.0563	4.4854

Table 5: Adjusted Markowitz Portfolio Statistics

Adjusted Shrinkage Portfolio Statistics							
T	63	126	252	756	1260	2520	7056
Return	0.0487	0.4263	0.3228	0.2385	0.2371	0.2517	0.2948
Volatility	0.4964	0.1416	0.0830	0.0556	0.0593	0.0574	0.0613
Sharpe	0.0579	2.8703	3.6498	3.9335	3.6633	4.0373	4.4854

Table 6: Adjusted Shrinkage Portfolio Statistics

Appendix 2 – Matlab Code

Simulated Data

DJIA Data Load

```
%DJIAdataload.m
%Summary: Load the Microsoft Excel files needed for the simulation of
%Multidimensional Geometric Brownian Motion
```

```
clear, clc
%DJIA_stats lists, by row, the starting price, annual return, and
%annual volatility for each stock in the DJIA
Stats = importdata('DJIA_stats.xlsx');
%DJIA_returns lists the daily returns from 1980-2014 for each stock in
%the DJIA
R = importdata('DJIA_returns.xlsx');
%Indicator of code completion
complete = 'complete'
```

Creation of Simulated Data

```
%MGBMcreator.m
%Summary: Uses the information from the data load to create 35 years of
%daily returns using Multidimensional Geometric Brownian Motion and
%plots the results
```

```
%Pulls the starting price from the summary statistics
start = Stats(1,:);
%Coverts the average annual returns to daily returns
Returns = [((1 + mean(R)).^252) - 1]';
%Creates the covariance matrix of the DJIA constituents
covmatrix = cov(R);
```

```
%Total time over which the simulation will occur
T = 35;
%Number of time steps within the simulation
N = 252*T;
%Size of each step
dt = T/N;
%Range of the simulation
t = 0:dt:T;
```

```
%Produces a matrix of random variables
B = randn(30,N);
%Creates the Cholesky Decomposition Matrix
C = chol(covmatrix);
V = C' * B;
```

```
%Creates a matrix to store the daily prices
Close = zeros(N + 1, 30);
```

```
%Loop will generate 35 years of daily prices for each of the 30 stocks
for p = 1:30
    price = start(:,p); %initial price of the pth stock
    mu = Returns(p,:); %average return of the pth stock
    sigma = sqrt(covmatrix(p,p))*sqrt(252); %volatility of the pth stock
```

```

%Generate the prices of the pth stock using MGBM
Simu = price * exp((mu - (sigma^2)/2)*t + sigma*[0,cumsum(V(p,:))]);

%Stores the prices of the pth stock in the daily prices matrix
Close(:,p) = [Simu]';

%Plot each stock simulation on the same graph
hold on
plot(t,Simu)
%Generates a legend to identify stocks by color
legendinfo{p} = [sprintf('Stock %f',p)];
end

legend(legendinfo);

```

Analysis of Simulated Data

Load Simulation Data

```

%LoadBase.m
%Summary: Load the data from CGBM simulation.

```

```

clear, clc
raw = importdata('RawData.xlsx');
raw = raw.data;
complete = 'complete'

```

Generate Returns

```

%GenerateReturns.m
%Summary: Calculate the data and effective annual returns for the
%simulated data.

```

```

%Find the size of the simulated data table
[R C] = size(raw);

```

```

%Set up matrix to fill with daily returns
ret = zeros(8820,30);

```

```

%The loop will calculate the daily return of each entry
for r = 1:(R-1)
    for c = 1:C
        reti = (raw(r+1,c)-raw(r,c))/raw(r,c);
        ret(r,c) = reti;
    end
end

```

```

%Compute compound return for each asset
ret1 = ret + 1;
compret = cumprod(ret1);
compret = compret((R-1),:);

```

```

%Compute effective annual return
effret = ((compret.^(1/35))-1);

```

```

complete = 'complete'

```


Plot Efficient Frontiers

```
%GeneratePortfolio.m
%Summary: The script is used to generate portfolios based on a specified
%value(s) of T. The weights of portfolio constituents was also pulled
%from this script.

%Set the different levels of T
T = [ 63 126 252 756 1260 2520 7560 ];
step = size(T,2);

%Create a matrix to hold the weights of the portfolios for each level of T
weights = zeros(30,step);

%Set count
count = 1;

%The loop will run for each value of T to generate an Efficient Frontier
%and a Capital Market Line
for t = T
    %Selects to amount of historical data available (aka T)
    sight = ret((7561-t):7560,:);

    %Calculate the compound return
    retsight = cumprod(sight + 1);
    cumret = retsight(t,:);
    mu = [ cumret.^(1/t) ]';

    %Decide between Markowitz and Shrinkage portfolio, and comment out the
    %line of the portfolio that will not be used
    sigma = [ cov(sight) ];
    %[Shrink sigma] = shrinkagecalc(sight); Shrink

    %Convert the risk free yield from annual to daily return
    r = (1.02)^(1/252);

    tlow = 0.; thigh = .1; tinc = 0.001;
    checkdata(sigma,1.e-8);
    [alpha0, alpha1, beta0, beta2, h0, h1] = EFMVcoeff(mu,sigma);
    [w] = CMLplot(tlow, thigh, tinc, alpha0, alpha1, beta0, beta2, mu, sigma,
r);
    weights(:,count) = w;
    count = count + 1;
end

grid on
axis([0 .03 .996 1.02])

complete = 'complete'
hold on
```

Compute & Plot Return, Volatility, and Sharpe Ratio

```
%PortfolioStats.m
%Summary: This script will be used to compute and plot the return,
%volatility, and Sharpe Ratio for all portfolios from 3 months to 10 years.

%Set the time range of T
T = 63:21:2520;
step = size(T,2);

%Create a table to hold the metrics: Row 1 = T, Row 2 = return,
%Row 3 = Volatility, Row 4 = Sharpe Ratio
Stats = zeros(4,step);
Stats(1,:) = T;
count = 1;

%Set the range of future prices that will be looked at
fut = 252;
Tfut = ret1(7561:(7560+fut),:);

%Calculate the compound return of the future prices
Rfut = cumprod(Tfut);
Rfut = Rfut(fut,:);

%Calculate the annual return for the future prices
AnnR = ((Rfut.^(252/fut))-1);

%This loop will generate the specified portfolios like GeneratePortfolio.m
%and then calculate the metrics for each T in the range provided
for t = T
    sight = ret((7561-t):7560,:);
    retsight = cumprod(sight + 1);
    cumret = retsight(t,:);
    mu = [ cumret.^(1/t) ]';

    %Decide between Markowitz and Shrinkage portfolio, and comment out the
    %line of the portfolio that will not be used
    sigma = [ cov(sight) ];
    %[Shrink sigma] = shrinkagecalc(sight);

    r = (1.02)^(1/252);
    tlow = 0.; thigh = .1; tinc = 0.001;
    checkdata(sigma,1.e-8);
    [alpha0, alpha1, beta0, beta2, h0, h1] = EFMVcoeff(mu,sigma);
    [w] = CMLcalc(tlow, thigh, tinc, alpha0, alpha1, beta0, beta2, mu, sigma,
r);

    %Generate the return, volatility, and Sharpe Ratio
    Stats(2, count) = AnnR * w;
    Stats(3, count) = sqrt(252)*sqrt(w'*sigma*w);
    Stats(4, count) = ((AnnR*w)-0.02)/(sqrt(252)*sqrt(w'*sigma*w));

    count = count + 1;
end
```

```

subplot(3,1,1)
plot(T,Stats(2,:), 'r')
ylabel('Annual Return')
axis([63 2520 -1 0.5])
grid on
hold on
subplot(3,1,2)
plot(T,Stats(3,:), 'r')
ylabel('Annual Volatility')
axis([63 2520 0 0.5])
grid on
hold on
subplot(3,1,3)
plot(T,Stats(4,:), 'r')
ylabel('Sharpe Ratio')
axis([63 2520 -2.5 6])
grid on
hold on

%Create a reference line for the annual values of the GT portfolio
subplot(3,1,1)
aline = reffline([0 0.298]);
aline.Color = 'g';
subplot(3,1,2)
bline = reffline([0 0.059]);
bline.Color = 'g';
subplot(3,1,3)
cline = reffline([0 4.695]);
cline.Color = 'g';

Active Share
%ActiveShare.m
%Summary: This script will be used to compute Active Share for the
%Markowitz and Shrinkage portfolios with a benchmark index of the GT
%portfolio.

T = 63:21:2520;
step = size(T,2);
Wnorm = zeros(30,step);
Wshrink = zeros(30,step);
count = 1;

%Weights of the GT portfolio
WGT = [0.512246612328401;-0.0464180325708238;-
0.171144972759123;0.0669926490742206;0.0733712463951512;0.229323119842787;-
0.0846446802417604;-0.139368880251176;-0.00458175123082576;-
0.232149888683918;-0.0525454616815763;-0.238506061264722;0.0223600681895427;-
0.0869593897054065;-
0.0403113447226787;0.064002530099778;0.00677824761679653;0.80089068873052;0.0
326767405857577;0.336785610642025;-0.174409415580255;-
0.0706245473272977;0.212740661687166;-0.260560668900181;-
0.00618625603970899;0.0255056763454046;0.112492998083287;0.138589776717328;0.
0342109079157655;-0.0605561832944767];
WGT = WGT*ones(1,118);

for t = T

```

```

sight = ret((7561-t):7560,:);
retsight = cumprod(sight + 1);
cumret = retsight(t,:);
mu = [ cumret.^(1/t) ]';
sigma = [ cov(sight) ];

r = (1.02)^(1/252);
tlow = 0.; thigh = .1; tinc = 0.001;
checkdata(sigma,1.e-8);
[alpha0, alpha1, beta0, beta2, h0, h1] = EFMVcoeff(mu,sigma);
[w] = CMLcalc(tlow, thigh, tinc, alpha0, alpha1, beta0, beta2, mu, sigma,
r);
Wnorm(:,count) = w;
count = count + 1;
end

count = 1;

for t = T
sight = ret((7561-t):7560,:);
retsight = cumprod(sight + 1);
cumret = retsight(t,:);
mu = [ cumret.^(1/t) ]';
[Shrink sigma] = shrinkagecalc(sight);

r = (1.02)^(1/252);
tlow = 0.; thigh = .1; tinc = 0.001;
checkdata(sigma,1.e-8);
[alpha0, alpha1, beta0, beta2, h0, h1] = EFMVcoeff(mu,sigma);
[w] = CMLcalc(tlow, thigh, tinc, alpha0, alpha1, beta0, beta2, mu, sigma,
r);
Wshrink(:,count) = w;
count = count + 1;
end

%Computation of Active Share
ASm = cumsum(abs(Wnorm - WGT))/2;
ASm = [ASm(30,:)];
ASs = cumsum(abs(Wshrink - WGT))/2;
ASs = [ASs(30,:)];

plot(T,ASm, 'b')
title('Active Share vs. GT Prortfolio')
xlabel('Time (days)')
ylabel('Active Share')
grid on
hold on
plot(T,ASs, 'r')
axis([0 2520 0 22])

complete = 'complete'

```